Confidential

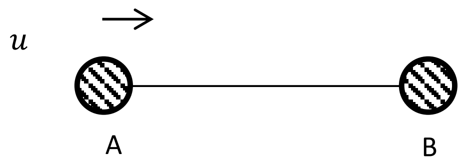
Ministry of Education

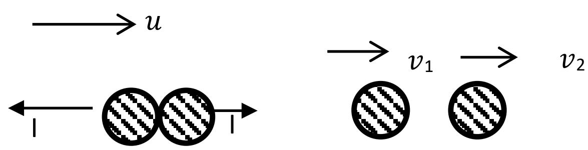
Support Seminar Paper-2023

10- Combined Mathematics II

Marking Scheme

01.





$\overrightarrow{I = \Delta MV}$ *for the system*

$0 = m(v_1 - u) + m(v_2 - 0)$ (05)

$v_1 + v_2 = u$ ——— (1)

$v_1 - v_2 = e(u - 0)$ (05)

$v_1 - v_2 = eu$ ——— (2)

(1) + (2) $\Longrightarrow v_2 = \frac{u}{2}(1 + e)$

(1) - (2) $\Longrightarrow v_1 = \frac{u}{2}(1 - e)$

$\rightarrow v_1 \rightarrow v_2 \quad \longrightarrow v_3 \longrightarrow v_3$

For system $A :\rightarrow I = \Delta mv$ (05)

$0 = m(v_3 - v_1) + m(v_3 - v_2)$

$v_3 = \frac{v_1 + v_2}{2}$

$v_3 = \frac{\frac{u}{2}.2}{2}$

$= \frac{u}{2}$

For $A \quad I = \Delta mv \rightarrow$ (05)

$I = m(v_3 - v_1)$

$OR\ for\ partical\ B \leftarrow$

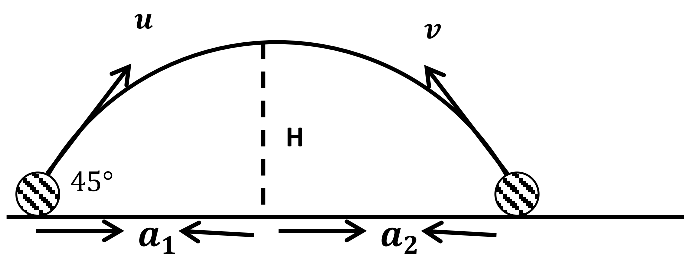
$I = m\left(-v_3 - (-v_2)\right)$

$= \frac{mue}{2}$ (05)

$= m\left(\frac{u}{2} - \frac{u}{2}(1-e)\right)$

$= \frac{mue}{2}$

Q2



For motion upto B

$A \uparrow \quad v = u + at$ for B $\uparrow v = u + at$ (05)

$0 = u \sin 45° - gt$ $0 = v \cos 60° - gt$

$t = \frac{u \sin 45°}{g}$ (05) $t = \frac{v \cos 60°}{g}$

By equating t

$\frac{u \sin 45°}{g} = \frac{v \cos 60°}{g}$ (05)

$\frac{u}{\sqrt{2}} = \frac{\sqrt{3}v}{2}$

$\frac{u}{v} = \frac{\sqrt{3}v}{\sqrt{2}}$

$\therefore \quad u : v = \sqrt{3} : \sqrt{2}$

$a_1 + a_2 = a$ (05)

$\frac{u \cos 45°.u \sin 45°}{g} + \frac{v \cos 60°.v \sin 60°}{g} = a$

$\frac{u^2}{2g} + \frac{\sqrt{3}v^2}{4\,g} = a$

$2u^2 + \sqrt{3}v^2 = 4ag$

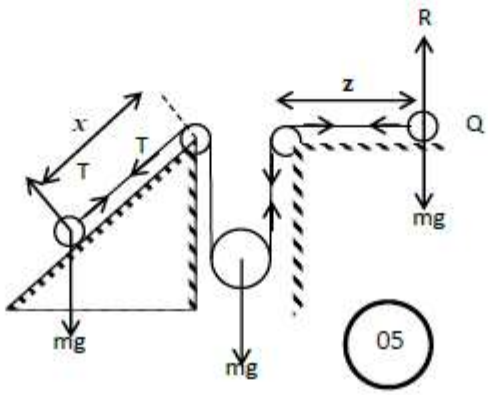
$2u^2 + \sqrt{3}\,\frac{2}{3}\,u^2 = 4ag$

$6u^2 + 2\sqrt{3}\, u^2 = 12ag$

$u^2 = \frac{6\, ag}{(3+\sqrt{3})} = \frac{2\sqrt{3}\;\; ag}{\sqrt{3}\; +1}$ 05

$u = \sqrt{\frac{2\,\sqrt{3}\;\;\; ag}{\sqrt{3}+1}}$

3)



05

$x + 2y + z = l$

$\ddot{x} + 2\ddot{y} + \ddot{z} = 0$

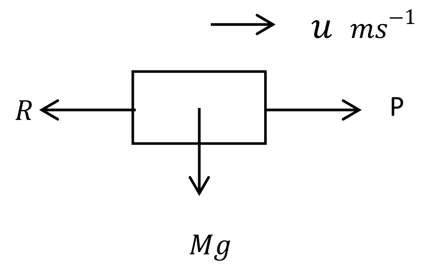
$\ddot{y} = -\frac{(\ddot{x}+\ddot{z})}{2}$ ——— (1) 05

For $P \swarrow \; mg \sin\propto \; - T = m\ddot{x}$ ——— (2) 05

For $Q \leftarrow T = -m\ddot{z}$ ——— (3) 05

For $R \downarrow \; M - 2T = M\ddot{y}$ ——— (4) 05

04.



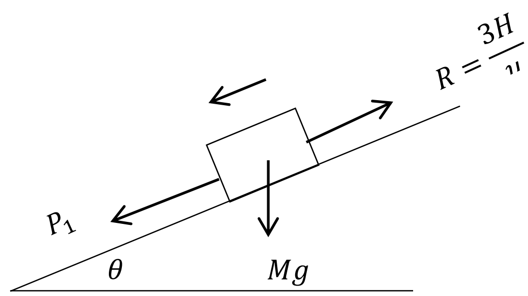
$P - R = M \times O$

$P = R$

$Pu = 3H \Rightarrow Ru = 3H$

$R = 3H \Rightarrow Ru = 3H$

$$R = \frac{3H}{u} N$$ (5)



$\sin\theta = \frac{1}{30}$

$P_1 - \mathrm{R} + \mathrm{Mg}\sin\theta = M \times O$ (10)

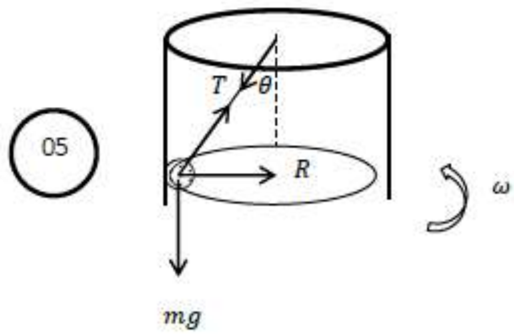
$P_1 = \mathrm{R} - \mathrm{Mg} \times \frac{1}{30} = \frac{3H}{u} - \frac{Mg}{30}$ (5)

$P_1\mathrm{V} = 3\mathrm{H}$ (5)

$\left(\frac{3H}{u} - \frac{Mg}{30}\right)\mathrm{V} = 3\mathrm{H}$

$$V = \frac{3H \times 30u}{90H - Mgu} \; ms^{-1}$$

(5)



m

$\uparrow F = ma$

$\uparrow T\cos\theta = mg$

$T = \frac{2mg}{\sqrt{3}}$ (05)

$Sin\theta = \frac{a}{2a} = \frac{1}{2}$

$\theta = \frac{\pi}{6}$ (05)

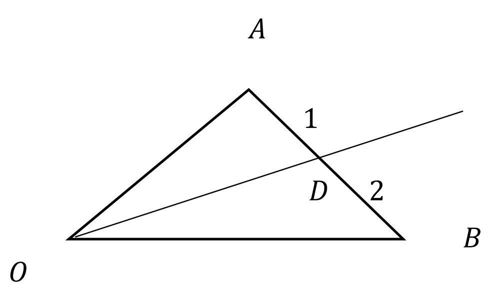
$\rightarrow F = ma$

$R + T\sin\theta = m.a\omega^2$ (05)

$R = ma\omega^2 - \frac{2mg}{\sqrt{3}} \cdot \frac{1}{2}$

$R = m\frac{(\sqrt{3}a\omega^2 - g)}{\sqrt{3}}$ (05)

06.



$\overrightarrow{OD} = \frac{2\times\overrightarrow{OA} + 1\times\overrightarrow{OB}}{2+1}$ (5)

$\overrightarrow{OD} = \frac{2(\underline{i}+\underline{j}) + 1(4\underline{i}+\underline{j})}{3}$

$\overrightarrow{OD} = \frac{6\underline{i}+3\underline{j}}{3}$

$\overrightarrow{OD} = 2\underline{i} + \underline{j}$ (5)

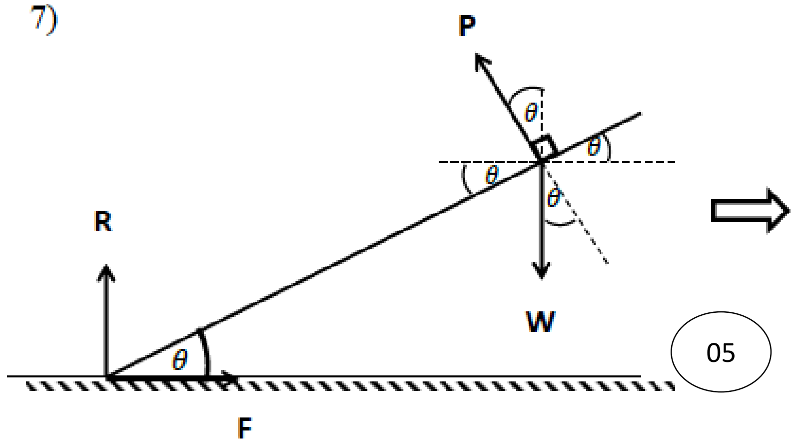
$\overrightarrow{OC} = 6\underline{i} + 3\underline{j} = 3(2\underline{i} + \underline{j})$ (5)

$\overrightarrow{OC} = 3\overrightarrow{OD}$ (5)

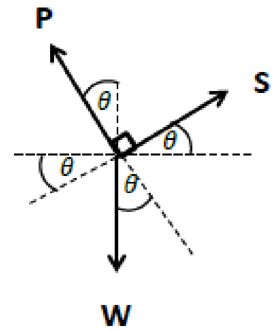
$\therefore OC \parallel OD (\because point\ O\ is\ common)$

$\therefore O, C\ and$ D Collinear. (5)

7)



05



$$\frac{S}{\sin(\pi-\theta)} = \frac{S}{\sin\left(\frac{\pi}{2}+\theta\right)} = \frac{S}{\sin\left(\frac{\pi}{2}\right)}$$ 10

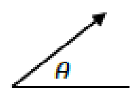
$$\frac{S}{\sin\theta} = \frac{P}{\cos\theta} = W$$

05

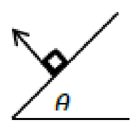
$S = W\sin\theta$

$P = W\cos\theta$

By considering the equibbrium



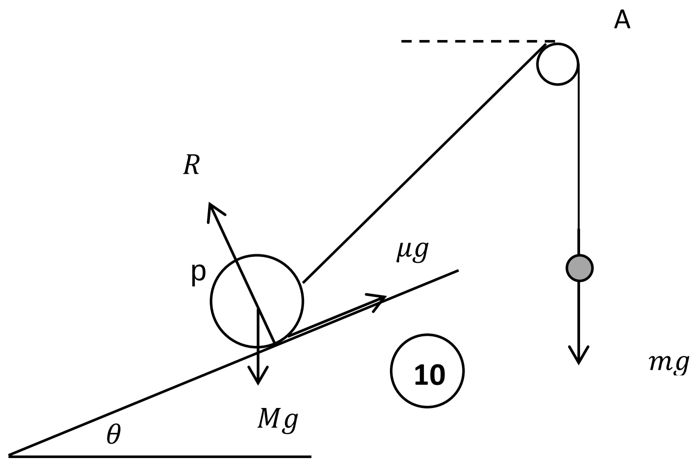
$S = W\sin\theta$ 05



$P = W\cos\theta$

05

8.



P අංශුවට $\longrightarrow$ $\mu R\cos\theta + T\sin\theta - mg\sin\theta = 0$

(5)

$Q \downarrow mg - T = 0$ (5)

$T = mg$

$P \uparrow R\cos\theta + T\cos\theta + \mu R\sin\theta - Mg = 0$ (5)

Q (9) $\because$ $P(A/B) = P(B/C) = 0$

$P(A \cap B) = \emptyset$ *and* $P(B \cap C) = \emptyset$

$\because A, B$ *mutually exclusive and* B, C *mutually exclusive.*

$A \cap B \cap C = (A \cap C) \cap B = \emptyset$ (05)

$P(A \cap B \cap C) = 0$

$\because$ $P(A/C) = P(A) = P(A \cap C) = P(A).P(C) = 3k^2$ (05) (05)

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= 3k + 2k + k - 0 - 0 - 3k^2 + 0$

$\frac{11}{12} = 6k - 3k^2$ (05)

$\therefore$ $36k^2 - 72k + 11 = 0$

$(6k - 1)(6k - 11) = 0$

$\because$ $6k - 11 \neq 0$, $k = \frac{1}{6}$ (05)

Q (10)

x	-2	-1	0	1	2
f	4	1	3	1	1
fx	-8	-1	0	1	2
fx^2	16	1	0	1	4

05

$\overline{x} = \frac{\sum fx}{\sum f} = -\frac{6}{10} = -0.6$

05

$\sigma^2 x = \frac{\sum fx^2}{\sum f} - \overline{x}^2 = \frac{22}{10} - 0.36 = 1.84$

05

$Let\ y = 2000 - 4x$

$Then\ \overline{y} = 2000 - 4\overline{x} = 2000 + 2.4$

05

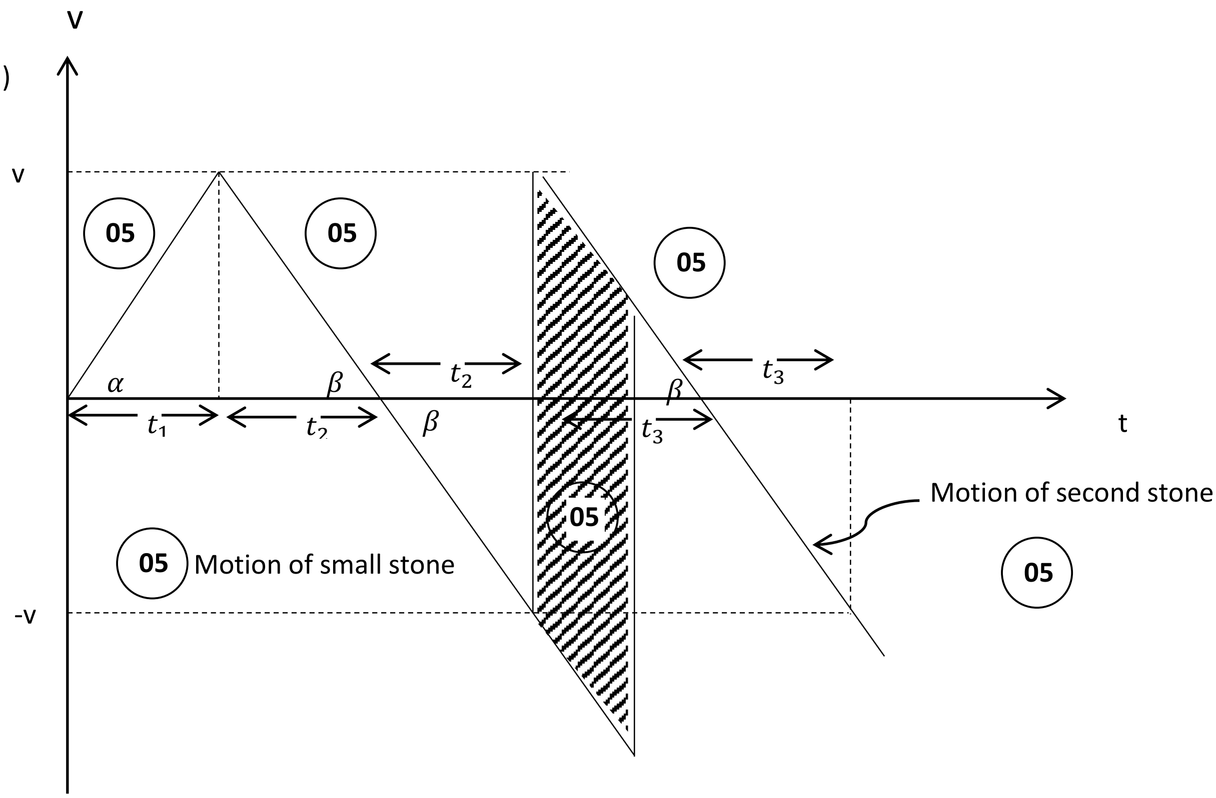
$= 2002.4$

$\sigma^2 y = 4^2 \sigma^2 x = 16 \times 1.84$

$\sigma y = \nless \sqrt{1.84}$

05

Q.11(a)



(ii) For the motion of small stone

$tan\alpha = \lambda g = \frac{v}{t_1} \Rightarrow v = \lambda g t_1$ 05

$H = \frac{1}{2} t_1 v$

$H = \frac{1}{2}\frac{v}{\lambda g} v \ \Rightarrow v = \sqrt{2\lambda gh}$ 05

(iii) Height attained by particle small

$H + \frac{1}{2} v t_2 \ where\ g = \frac{v}{t_2} \Rightarrow t_2 = \frac{v}{g}$ 05

$H + \frac{1}{2}.v.\frac{v}{g}$ 05

$H + \frac{v^2}{2g}$

(iv) Time taken to collide is equal to t

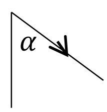
$\frac{1}{2}(2v+2v).t=h$ (05)

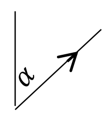
$t=\frac{h}{2v}$

$=\frac{h}{2\sqrt{2\lambda gh}}$ (05)

$=\sqrt{\frac{h}{8\lambda g}}$

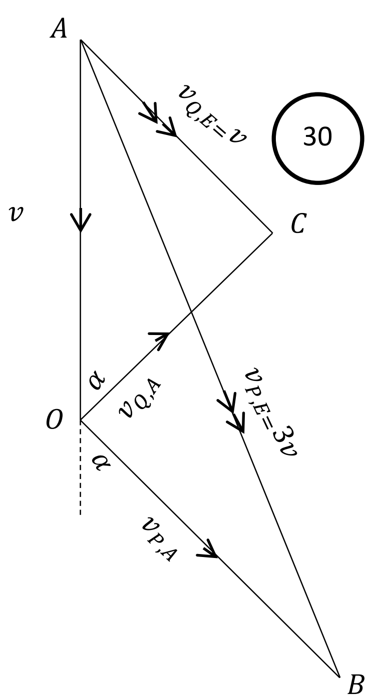
Question 11 (b)

$v_{A,\ E}=$ ↓ v $v_{P\ \ A}=$ 

$v_{Q\ \ A}=$ 

$v=\ 3v$ $v_{Q,\ \ E}=v$ --------------(5)

By Relative velocity principle



$v_{P,E}=v_{P,A}+v_{A,E}$
$3v=$ ↘ + v ↓ ----------------(5)
$v_{Q,E}=v_{Q,A}+v_{A,E}$
$v=$↗ +↓ v ----------------(5)

OAB *for the motions of* A *and* P

OAC *for the motions of* A *and* Q

Let $v_{P,A}=v_1$
$v_{Q,A}=v_2$

$(v+v_1cos\alpha)^2+(v_1sin\alpha)^2=(3v)^2$
${v_1}^2+2vv_1cos\alpha+v^2=9v^2$ ---------------(5)
${v_1}^2+2vv_1cos\alpha+v^2cos^2\alpha+v^2sin^2\alpha=9v^2$
$(v_1+vcos\alpha)^2=9v^2-v^2sin^2\alpha$

$v_1 + vcos\alpha = v\sqrt{9 - sin^2\alpha}$ ---------------(5)

$v_1 = v(\sqrt{9 - sin^2\alpha} - cos\alpha)$

$v_{P,A} = v(\sqrt{9 - sin^2\alpha} - cos\alpha)$ --------------(5)

$(v - v_2cos\alpha)^2 + v_2sin^2\alpha = v^2$

$v^2 + {v_2}^2 - 2vv_2cos\alpha = v^2$

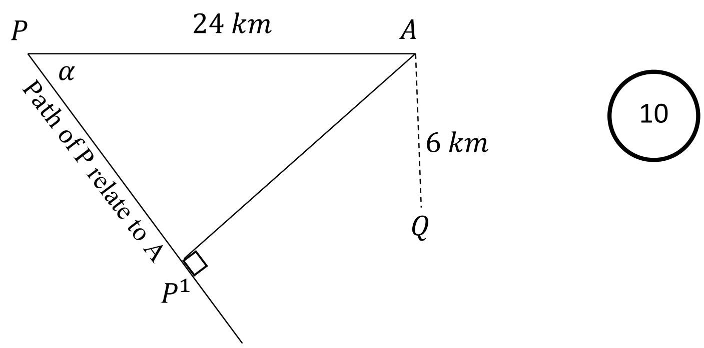
$(v_2 - vcos\alpha)^2 + v^2sin^2\alpha = v^2$

$v_2 = v\{\sqrt{1 - sin^2}\alpha + cos\alpha\}$

$v_{Q,A} = v\{\sqrt{1 - sin^2}\alpha + cos\alpha\}$ ---------(5)

$v_{Q,A} = 2vcos\alpha$

(ii)



$cos\alpha = \dfrac{PP^1}{24}$

$PP^1 = 24cos\alpha$ ------------(5)

$\text{Time taken} = \dfrac{\text{Distance travelled by P relatedto A}}{v_{P,A}}$

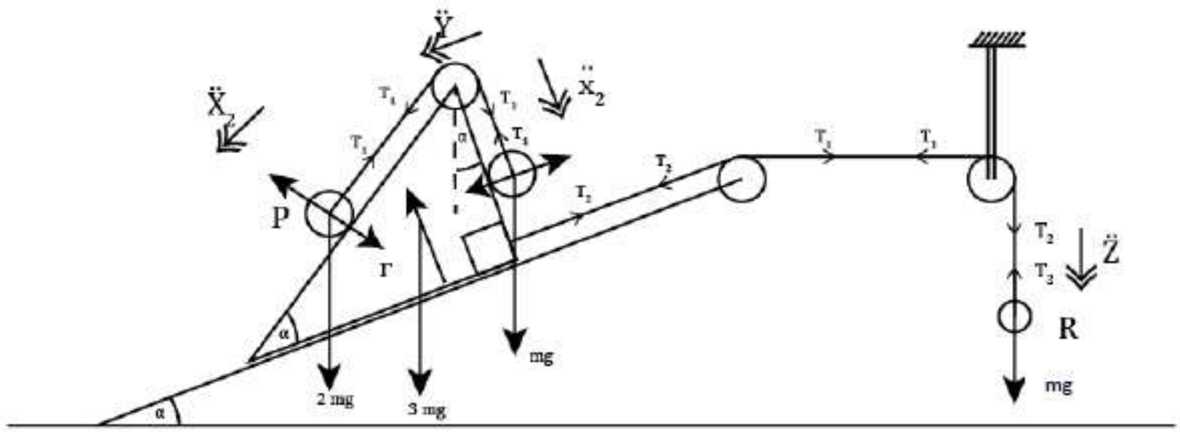
$= \dfrac{24cos\alpha}{v\{\sqrt{9 - sin^2\alpha} - cos\alpha\}}$ --------------------(5)

Distance travelled by Q at this time = $v_{P,A} \times t$

$= 2vcos\alpha \times \dfrac{24cos\alpha}{v\{\sqrt{9 - sin^2\alpha} - cos\alpha\}}$

$= \dfrac{48cos^2\alpha}{\sqrt{\{\sqrt{9 - sin^2\alpha} - cos\alpha\}}}$ ---------------------------(5)

(12) a

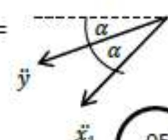
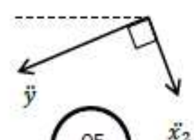


$x_1 + x_2 = l_1$

$\ddot{x}_1 + \ddot{x}_2 = 0$ —— (1) (05)

$y + z + k = l_2$

$\ddot{y} + \ddot{z} = 0$ —— (2) (05)

$a_{YE} = \swarrow \ddot{y}$ $\quad a_{RE} = \downarrow \ddot{z}$ $\quad a_{PE} =$  (05) $\quad a_{QE} =$  (05)

$F = ma$

For $P \swarrow 2mg \sin 2\propto - T_1 = 2m(\ddot{x}_1 + \ddot{y}\cos\propto)$ —— (3) (10)

For $Q_1 \searrow mg\cos\propto - T_2 = m(\ddot{x}_2)$ —— (4) (10)

For $R_1 \downarrow mg - T_2 = m(\ddot{z})$ —— (5) (10)

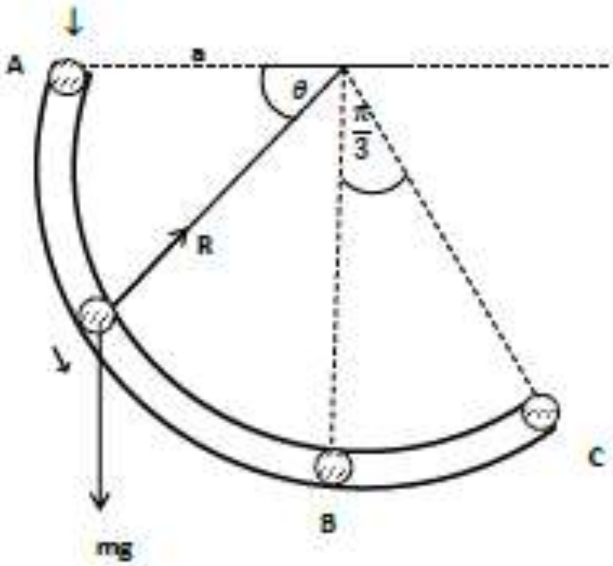
For the system P and Q

$T_2 - 6mg\sin\propto = 2m(-\ddot{y} - \ddot{x}_1\cos\propto) = 3m(\ddot{y}) + m(-\ddot{y})$ —— (6) (20)

For $\quad R \downarrow s = ut + \frac{1}{2}at^2$

$a = 0 + \frac{1}{2}\ddot{z}t^2$ —— (7) (05)

(12) b



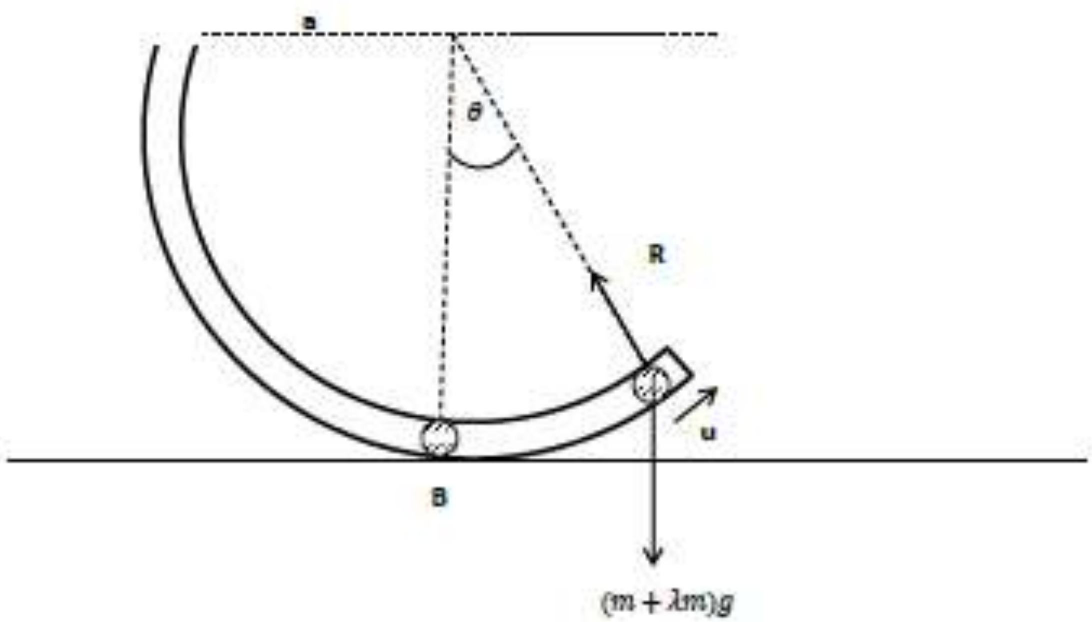
05

$0 = \frac{1}{2}mv^2 - mga\sin\theta$ 05

$v^2 = 2ga\sin\theta$ 05

$v_1^2 = 2ga\sin\frac{\pi}{2}$

$v_1 = \sqrt{2ga}$ 05



$\rightarrow v_1 \rightarrow 0 \qquad\qquad \rightarrow v_2 \ \rightarrow v_2$

$\rightarrow I = \Delta mv$

$0 = (m + \lambda m)v_2 - mv_1$ (05)

$v_2 = \frac{\sqrt{2ga}}{(1+\lambda)}$ (05)

$\frac{1}{2}(m + \lambda m)v_2^2 = \frac{1}{2}(m + \lambda m)u^2 + (m + \lambda m)g(a - a\cos\theta)$ (15)

$\frac{2ga}{2(1+\lambda)^2} = \frac{u^2}{2} + ga(1 - \cos\theta)$

$u^2 = \frac{2ga}{(1+\lambda)^2} - 2ga(1 - \cos\theta) = 2ga\left(\frac{1}{(1+\lambda)^2} + \cos\theta - 1\right)$

$when\ u = 0\ ,\ \ \theta = \propto$

$\frac{1}{(1+\lambda)^2} + \cos\propto - 1 = 0$

$\cos\propto = 1 - \frac{1}{(1+\lambda)^2}$ (05)

$when\ \propto < \frac{\pi}{3}$ (05)

$\cos\alpha > \cos\frac{\pi}{3}$

$1 - \frac{1}{(1+\lambda)^2} > \frac{1}{2}$

$\frac{1}{(1+\lambda)^2} < \frac{1}{2}$

$(1 + \lambda)^2 > 2 \rightarrow$

$1 + 2\lambda + \lambda^2 > 2$

$\lambda(\lambda + 2) > 1$ (05)

$\lambda = \sqrt{2} - 1$

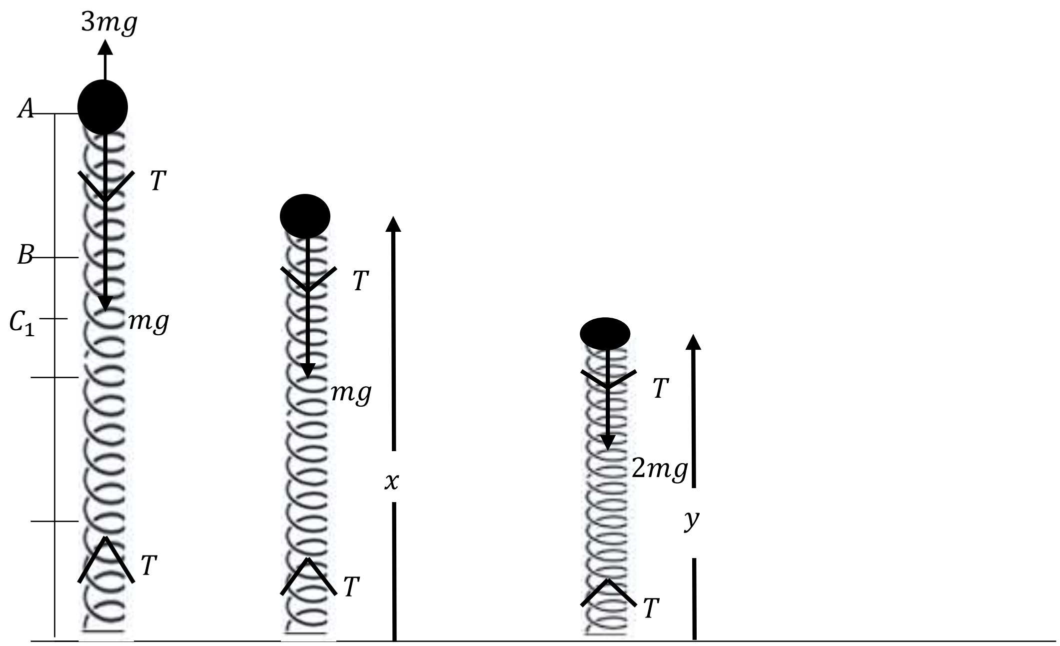
$\nwarrow F = ma$

$R - (m + \lambda m)g\sin\propto = (m + \lambda m)\frac{v^2}{a}$ (05)

$R = m(1 + \lambda)g\sin\frac{\pi}{3}$

$R = \sqrt{2}mg \times \frac{\sqrt{3}}{2} = \sqrt{\frac{3}{2}}mg$ (05)

13.



When the particle at A

$\uparrow 3mg - mg - T = 0$

$2mg = \frac{\lambda l}{3l}$ (05)

$\lambda = 6mg$

15

When the partical lies between at B

$\uparrow\ -mg - T = m\ddot{x}$ (05)

$-mg - \frac{6mg(x-3l)}{3l} = m\ddot{x}$ (10)

$\ddot{x} = -\frac{2g}{l}\left(x - \frac{5l}{2}\right)$ (05)

$\ddot{x} = -\omega^2 X$ (05)

$At\ center\ X = 0 \ \ \leftrightarrow x = \frac{5l}{2}$

$\dot{x} = \dot{x} - \frac{5l}{2}$ (10)

$\ddot{X} = \ddot{x}$ (05)

30

$At\ A\ x = \frac{3l}{2}, \dot{x} = 0$

$\dot{x}^2 = \omega^2(C^2 - X^2)$ (05)

$0 = \omega^2\left(C^2 - \left(\frac{3l}{2}\right)^2\right)$ $\therefore Amplitude\ C = \pm\frac{3l}{2}$ (05)

$At\ B\ \dot{X}_B{}^2 = \frac{2g}{l}\left(\frac{9l^2}{4} - \left(\frac{l}{2}\right)^2\right)$ $\dot{X}_B = 2\sqrt{gl}$ (05)

15

$After\ Collision\ \downarrow I = \Delta mv$

$0 = m\left(+v - 2\sqrt{gl}\right) + m(v - 0)$ (05)

$V_B = \sqrt{gl}$ (05)

10

$When\ the\ partical\ lies\ between\ B\&D$

$\uparrow\ \underline{f} = m\underline{a}$

$T - 2mg = 2m\ddot{y}$ (05)

$\frac{6mg(3l-y)}{3l} - 2mg = 2m\ddot{y}$

(05)

$\ddot{y} = -\frac{g}{l}(y - 2l)$ (05)

15

$At\ the\ center\ \ddot{y} = 0 \qquad y = 2l$

$At\ B; t = 0, y = 3l\ , \dot{y} = \sqrt{gl}$

$y = 2l + \alpha\cos\omega t + \beta\sin\omega t$

(05) (05)

$3l = 2l + \alpha \quad \leftrightarrow \quad \alpha = l$

$\dot{y} = -\alpha\omega \sin \omega t + \beta\omega \cos \omega t$ (05)

$-\sqrt{gl} = \beta\omega$ (05)

$\ddot{y} = -\alpha\omega^2 \cos \omega t - \omega^2 sin\omega t$ (05)

$= -\omega^2(y - 2l)$ (05)

$\therefore \omega = \sqrt{\frac{g}{l}}$ $\quad \beta = -l$

(05) (05)

40

$At\ the\ end\ of\ the\ amplitude\ \dot{y} = 0$

$\tan \omega t = -1$ (05) $\leftrightarrow$ $\omega t = \frac{3\pi}{4}$ (05)

$y = 2l + lcos\frac{3\pi}{4} - lsin\frac{3\pi}{4}$ (05)

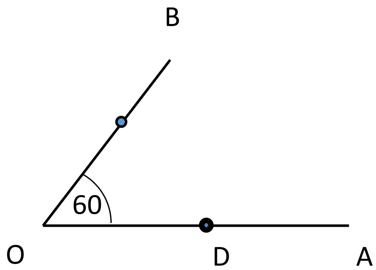
$= 2l - \frac{l}{\sqrt{2}} - \frac{l}{\sqrt{2}}$

$= 2l - \sqrt{2}l \quad \therefore amplitude = \sqrt{2}l$

(05) (05)

25

14. 14.I



A($\sqrt{3}\underline{i} + \underline{j}$)

$\overrightarrow{OA}.\overrightarrow{OB} = \left(\sqrt{3}\underline{i} + \underline{j}\right).(\alpha\underline{i} + \beta\underline{j})$ 05

05 $\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right| \cos 60° = \sqrt{3}\alpha + \beta$ 05

$\sqrt{3 + 1^2} \times 10 \times \frac{1}{2} = \sqrt{3} + \beta$

$10 = \sqrt{3}\alpha + \beta$ 05

$\overrightarrow{OB} = 5\sqrt{3}\underline{i} - 5\underline{j}$

$\alpha^2 + \beta^2 = 10^2$ 05

$\alpha^2 + \left(10 - \sqrt{3}\alpha\right)^2 = 100$

$\alpha^2 + 100 + 3\alpha^2 - 20\sqrt{3}\alpha = 100$

$4\alpha^2 - 20\sqrt{3}\alpha = 0$

$\alpha\left(\alpha - 5\sqrt{3}\right) = 0$

$\because \alpha \neq 0$

$\alpha = 5\sqrt{3}$ 05

$\beta = -5$ 05

35

$OC : CB = 1 : \lambda$

$\overrightarrow{OC} = \frac{1}{\lambda+1} \quad \overrightarrow{OB} = \frac{1}{\lambda+1}(5\sqrt{3}\underline{i} - 5\underline{j})$

05

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$ 05

$\frac{\sqrt{3}\underline{i}}{2} - \frac{5\underline{j}}{2} = -\sqrt{3}\underline{i} - \underline{j} + \frac{1}{(\lambda + 1)}\left(5\sqrt{3}\underline{i} - 5\underline{j}\right)$ 10

$\frac{\sqrt{3}\underline{i}}{2} - \frac{5\underline{j}}{2} = -\sqrt{3}\underline{i} + \frac{5\sqrt{3}}{\lambda+1}\underline{i} - \underline{j} - \frac{5}{\lambda+1}\underline{j}$

Comparing Coefficients of : $\underline{i}, \underline{j}$

$\underline{i} \rightarrow \quad -\sqrt{3} + \frac{5\sqrt{3}}{\lambda + 1} = \frac{\sqrt{3}}{2}$

$-2 + \frac{10}{\lambda + 1} = 1$

$\frac{10}{\lambda + 1} = 3$

$10 = 3\lambda + 3$

$\lambda = \frac{7}{3}$ 05

$\underline{j} \rightarrow \quad -\frac{5}{2} = -1 - \frac{5}{\lambda + 1}$

$-5 = -2 - \frac{10}{\lambda + 1}$

$\frac{10}{\lambda + 1} = 3$

$10 = 3\lambda + 3$

$\lambda = \frac{7}{3}$ 05

$OC:CB = 1:\frac{7}{3}$

$OC:CB = 3:7$

05

35

$\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$

$= -5\sqrt{3}\underline{i} + 5\underline{j} + \frac{1}{2}\overrightarrow{OA}$

$= -5\sqrt{3}\underline{i} + 5\underline{j} + \frac{1}{2}(\sqrt{3}\underline{i} + \underline{j})$

$= \frac{1}{2}(-10\sqrt{3}\underline{i} + 10\underline{j} + \sqrt{3}\underline{i} + \underline{j})$

$\overrightarrow{BD} = \frac{-9\sqrt{3}\underline{i}}{2} + \frac{11\underline{j}}{2}$ **5**

$$\overrightarrow{AE} = \frac{10}{17}\overrightarrow{AC} = \frac{10}{17}(\overrightarrow{AO} + \overrightarrow{OC})$$ 5

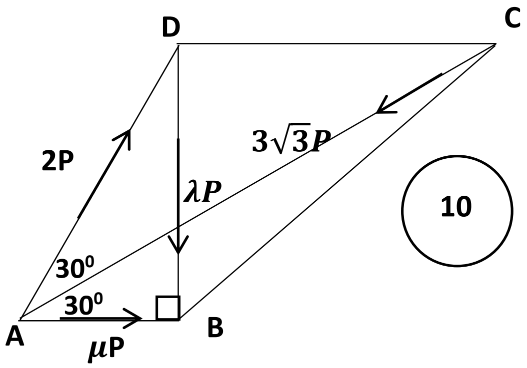
$$\overrightarrow{AE} = \frac{10}{17}\left(-\sqrt{3}\underline{i} - \underline{j} + \frac{3}{10}\left(5\sqrt{3}\underline{i} - 5\underline{j}\right)\right)$$

$$= \frac{10}{17}\left(-\sqrt{3}\underline{i} - \underline{j} + \frac{3}{10}\left(5\sqrt{3}\underline{i} - 5\underline{j}\right)\right)$$

$$= \frac{10}{17}\left(-\sqrt{3}\underline{i} - \underline{j} + \frac{3}{10}\left(5\sqrt{3}\underline{i} - 5\underline{j}\right)\right)$$ 5

$$= \frac{10}{17}\left(-\sqrt{3}\underline{i} - \underline{j} + \left(\frac{3\sqrt{3}\underline{i}}{2} - \frac{3\underline{j}}{2}\right)\right)$$

14.b



$$B^{\curvearrowleft} = -3\sqrt{3}P \times AB\ Sin\ 30 + 2P \times AB\ Sin\ 60$$ 10

$$B^{\curvearrowleft} = -3\sqrt{3}P \times \frac{1}{2} \times AB + 2P \times \frac{\sqrt{3}}{2} \times AB = -\frac{\sqrt{3}\,P}{2} \times AB$$

$$B^{\curvearrowleft} = \frac{\sqrt{3}\,P}{2} \times AB \neq 0$$ 5

∴ the sys from in not equilibrium Value of λ, μ

$A \curvearrowright = 0$

$\lambda P \times A\beta = 0$ 10

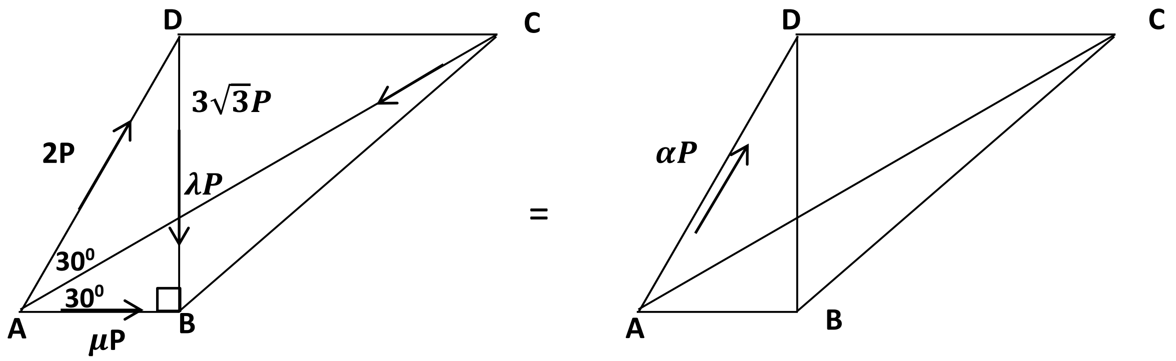
$P \neq O \quad AB \neq 0 \quad \lambda = O$

$D \curvearrowright = O$

- μP ×DB Sin 60 +$3\sqrt{3}$ P ×AD Sin 30^0 = 0 10

$\mu \times DB \times \frac{\sqrt{3}}{2} = 3\sqrt{3} \times AD \times \frac{1}{2}$; AD ≠ 0

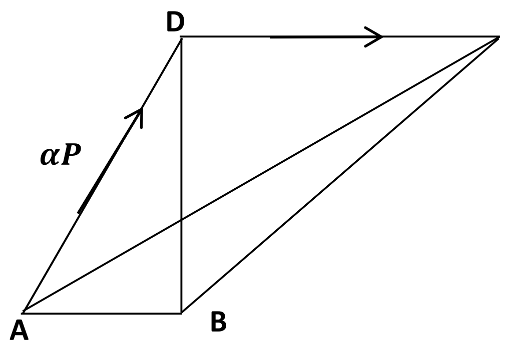
$\mu = 3$



$C \curvearrowleft = 3P \times AC \sin 30 - 2P \times AC \sin 30 = \alpha P \times AC \sin 30$ 10

$\frac{3}{2} - \frac{2}{2} = \frac{\alpha}{2}$

$\alpha = 1$ *if the line of action of new resultant* ……………………..

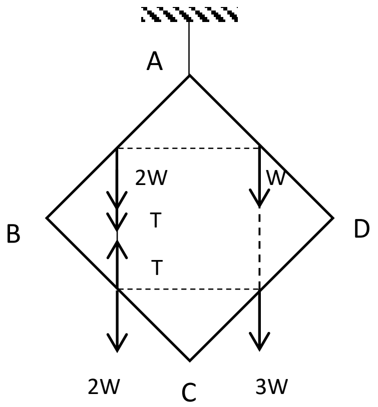


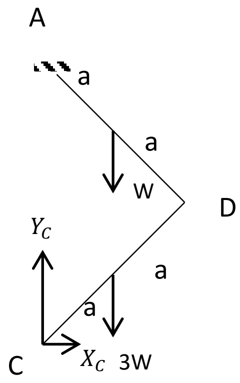
10

$G \circlearrowleft \qquad D = 0$

$G = 0$

15 a)





$A \circlearrowleft$

$4W\,.\,acos\frac{\pi}{4} = X_C.\,4acos\frac{\pi}{4}$ **(10)**

$X_c = W$ **(05)**

(10)

DC

$Y_C\,.2acos\frac{\pi}{4} = X_C.\,2acos\frac{\pi}{4} + 3W.\,acos\frac{\pi}{4}$

$\therefore Y_c = \frac{5}{2}W$ **(05)**

$\therefore\ C$

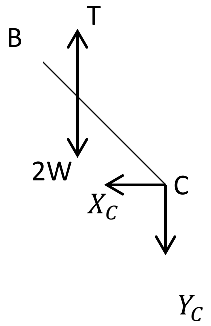
$R_C = \sqrt{{X_C}^2 + {Y_C}^2}$

$= \sqrt{W^2 + \frac{25}{4}W^2}$

$= \sqrt{W^2 + \frac{25}{4}W^2}$

$= \frac{\sqrt{29}}{2}W$ **(05)**

BC



(10)

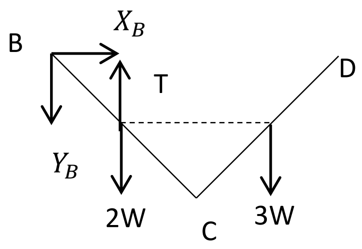
$B \curvearrowleft$

$$T.aCos\frac{\pi}{4} = 2W.aCos\frac{\pi}{4} + X_C.2acos\frac{\pi}{4} + Y_C.2acos\frac{\pi}{4}$$

$\therefore T = 2W + 2W + 5W$

$T = 9W$ (05)

BC+CD



$D \curvearrowleft$

$$Y_B.4aCos\frac{\pi}{4} + 2W.3aCos\frac{\pi}{4} + 3Wacos\frac{\pi}{4} = T.3acos\frac{\pi}{4}$$ (10)

$4Y_B + 6W + 3W = 3T = 3(9W)$

$\therefore Y_B = \frac{9}{2}W$ (05)

BC

$X_B = X_C = W$ (05)

$\therefore B$

$$R_B = \sqrt{W^2 + \frac{81}{4}W^2}$$

$= \frac{\sqrt{85}}{2}W$ (05)

A $\quad R_2.2a + (3W + W)acos\frac{\pi}{3} = 2W.a + (W + W)\left(2a + acos\frac{\pi}{3}\right)$ --------------(10)

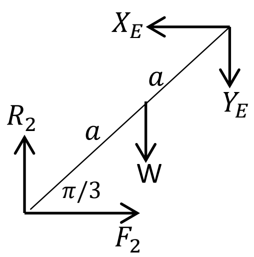
$R_2 = \frac{5}{2}W$ --------------(5)

$\uparrow$

$R_1 + R_2 = 8W$

$\therefore R_1 = \frac{11}{2}W$ -------------(5)

Only the rod EF,



↶ E

$$F_2.2a\cos\frac{\pi}{6} + W.a\cos\frac{\pi}{3} = R_2.2a\cos\frac{\pi}{3}$$ --------------(10)

$$F_2 = \frac{2}{\sqrt{3}}W$$ ----------------(5)

↑

$$R_2 = Y_E + W$$

$$Y_E = \frac{3}{2}W$$ -------------------(5)

→

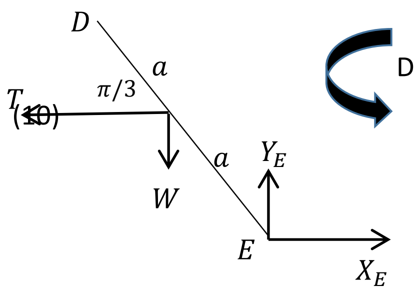
$$X_E = F_2 = \frac{2}{\sqrt{3}}W$$ -------------------(5)

Entire system

→

$$F_1 = F_2 = \frac{2}{\sqrt{3}}W$$ -------------------(5)

Only the rod DE



↶ D

$$T.a\cos\frac{\pi}{6} + W.a\cos\frac{\pi}{3} = Y_E.2a\cos\frac{\pi}{3} + X_E.2a\cos\frac{\pi}{6}$$ -----------(10)

$$T = 2\sqrt{3}W$$

For the equilibrium at A

$$F_1 \leq \mu R_1$$

$$\frac{2}{\sqrt{3}}W \leq \mu\frac{11}{2}W$$

$$\frac{4}{11\sqrt{3}} \leq \mu$$ ------------------(5)

For the equilibrium at F

$$F_2 \leq \mu R_2$$

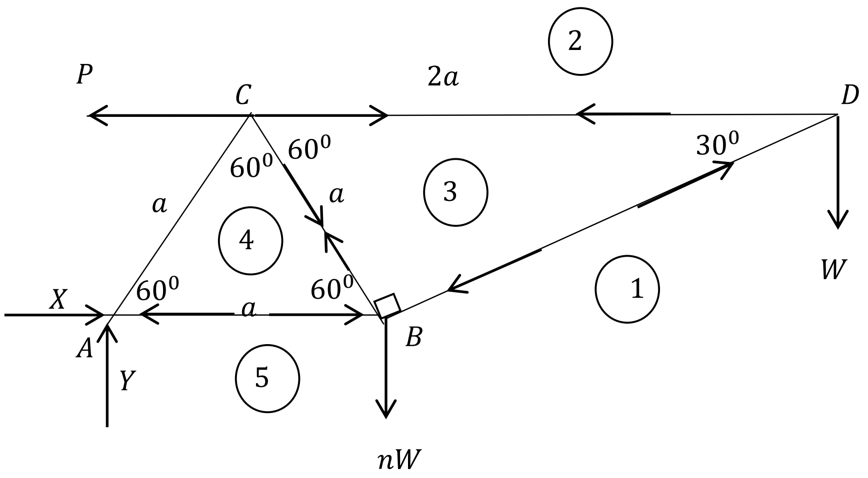
$$\frac{2}{\sqrt{3}}W \leq \mu \frac{5}{2}W$$

$\frac{4}{5\sqrt{3}} \leq \mu$-------------------(5)

$\therefore If\ \frac{4}{11\sqrt{3}} < \mu < \frac{4}{5\sqrt{3}}$ then -------------------(5)

Even though the point A is in equilibrium, the point F is not in equilibrium.

b)



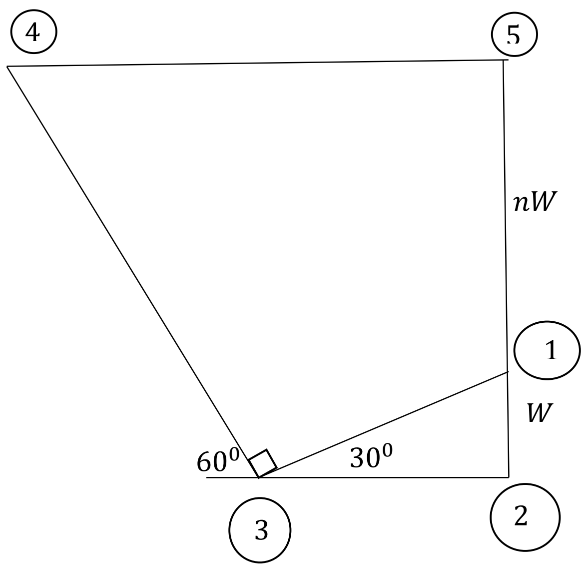
Lets take $AB = BC = AC = a$

$$CD = 2a$$

By considering the entire system,

A $\quad P.a\cos\frac{\pi}{6} = nWa + w\left(2a + \frac{a}{2}\right)$ -------------------------(10)

$$P = \left(\frac{2n+5}{\sqrt{3}}\right)W$$



Rod	Tension	Thrust
AB	______	$\left(\frac{n+4}{\sqrt{3}}\right)W$-------(5+5)
BC	$\frac{2}{\sqrt{3}}$(n+1)W-------(5+5)	______
CD	$\sqrt{3}W$-------(5+5)	______
BD	______	$2W$

$(1)(3)cos\frac{\pi}{3} = (1)(2)$ $\quad$ $(2)(3) = (1)(3)cos\frac{\pi}{6}$ $\quad$ $(3)(4)cos\frac{\pi}{6} = nW + W$

$(1)(3) = 2W$ $\quad$ $= \sqrt{3}W$ $\quad$ $(3)(4) = \frac{2}{\sqrt{3}}(n + 1)W$

$$(4)(5) = (2)(3) + (3)(4)cos\frac{\pi}{3}$$

$$(4)(5) = \sqrt{3}W + \frac{(n + 1)}{\sqrt{3}}W$$

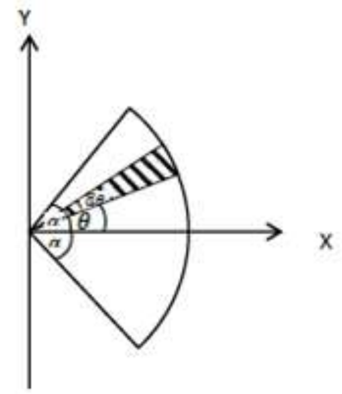
If the maximum possible tension for the rod BC is $10\sqrt{3}W$,

Then $\frac{2(n+1)}{\sqrt{3}}W \leq 10\sqrt{3}W$ -----------------(10)

$$n \leq 14$$

16.

(a) (1)



By the definition of center of mass.

$$\bar{x} = \frac{\int_{-\propto}^{+\propto} \frac{2}{3}r\cos\theta \frac{1}{2}r^2 d\theta\rho}{\int_{-\propto}^{\propto} \frac{1}{2}r^2 d\theta\, \rho}$$ (5) (5)

$$= \frac{\frac{1}{2}mr^2 \frac{2}{3}r\int_{-\propto}^{\propto}\cos\theta\, d\theta}{\frac{1}{2}mr^2\int_{-\propto}^{\propto} d\theta} \qquad = \frac{2}{3}r\frac{[\sin\theta]_{-\propto}^{+\propto}}{[\theta]_{-\propto}^{+\propto}}$$ (5)

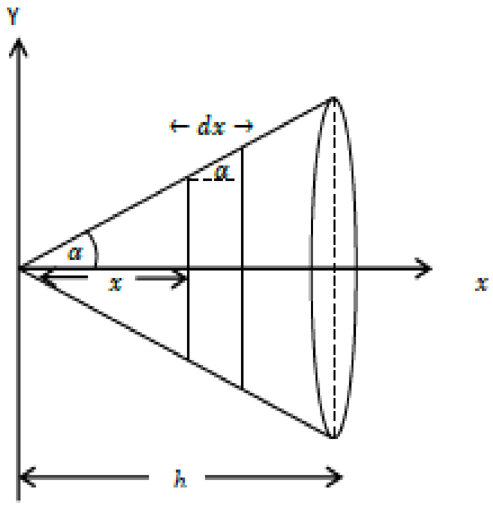
$$= \frac{2}{3}r\frac{[\sin\propto - \sin(-\propto)]}{[\propto-(-\propto)]} \qquad = \frac{2}{3}r\frac{2\sin\propto}{2\propto} \qquad = \frac{2r\sin\propto}{3\propto}$$ (5)

The center of mass of uniform secter lies on its symmetrical axis at a distance $\frac{2r\sin\propto}{3\propto}$ *from O.*

(5)

25

(a) (II)



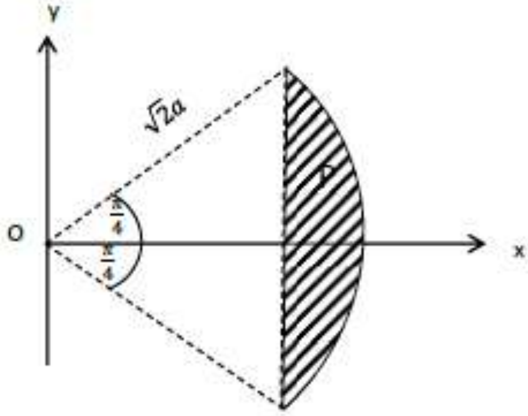
$$\bar{x} = \frac{\int_0^h 2\pi x \tan\alpha\, dx \sec\alpha\, \rho\, x}{\int_0^h 2\pi x \tan\alpha\, dx \sec\alpha\, \rho} \quad (5) \quad (5) \quad = \frac{2\pi \tan\alpha \sec\alpha\, \rho \int_0^h x^2}{2\pi \tan \sec \rho \int_0^h x}$$

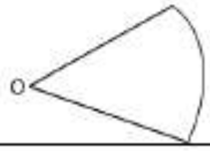
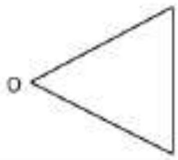
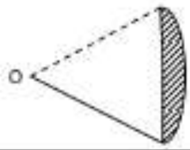
$$= \frac{\left[\frac{x^3}{3}\right]_0^h}{\left[\frac{x^2}{2}\right]_0^h} \quad (5)$$

$$= \frac{2}{3}h \quad (5)$$

The center of mass of uniform hollow cone lies on its symmetrical axis at a distance $\frac{2h}{3}$ *form O.* (5)

25



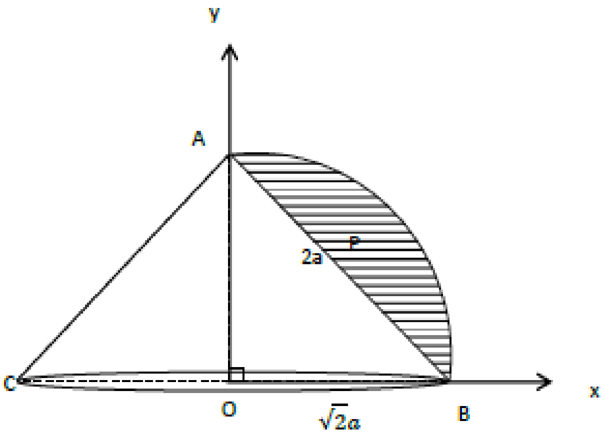
Object	Mass	$\bar{x}$
	$\frac{\pi a^2}{2}\rho$ (5)	$\frac{8a}{3\pi}$ (5)
	$a^2\rho$ (5)	$\frac{2}{3}a$ (5)
	$a^2\left(\pi/2 - 1\right)\rho$ (5)	$\bar{x}$

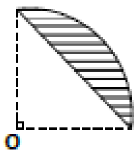
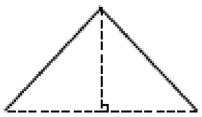
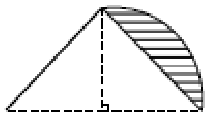
$$\bar{x} = \frac{\frac{\pi a^2}{2}\rho\frac{8a}{3\pi} - a^2\rho\frac{2}{3}a}{a^2\left(\pi/2 - 1\right)} = \frac{\frac{4a}{3} - \frac{2a}{3}}{\pi/2 - 1} = \frac{4a}{3(\pi - 2)}$$ (5)

(5)

Centre of gravity of the object is on the ox symmetric axis as it is symmetric about ox

40



Object	Mass	$\bar{x}$	$\bar{y}$
	M	$\frac{2\sqrt{2}a}{3(\pi-2)}$ (5)	$\frac{2\sqrt{2}a}{3(\pi-2)}$ (5)
	5M	$\frac{\sqrt{2}a}{3}$ (5)	0 (5)
	6M	$\bar{x}$	$\bar{y}$

$6M\bar{x} = \frac{M\times2\sqrt{2}a}{3(\pi-2)} + 5M\times\frac{\sqrt{2}}{3}a$

$\bar{x} = \frac{2\sqrt{2}a+5\sqrt{2}a(\pi-2)}{18(\pi-2)}$ (10)

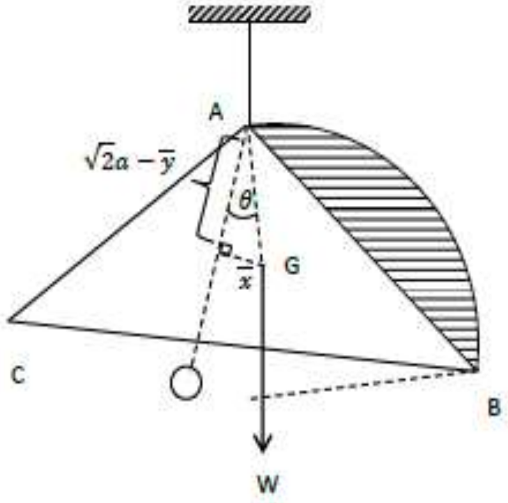
$= \frac{2+5(\pi-2)2\sqrt{2}a}{18(\pi-2)}$

$= \frac{(5\pi-2)\sqrt{2}a}{18(\pi-2)}$

$6M\bar{y} = \frac{M2\sqrt{2}a}{3(\pi-2)}$

$= \frac{\sqrt{2}a}{9(\pi-2)}$ (10)

40



(5)

$$\tan\theta = \frac{\bar{x}}{\sqrt{2}a - \bar{y}}$$ (5)

$$= \frac{\frac{a(5\pi-8)\sqrt{2}a}{18(\pi-2)}}{\sqrt{2}a - \frac{\sqrt{2}a}{9(\pi-2)}}$$ (5)

$$= \frac{(5\pi-8)\sqrt{2}a}{2[9\sqrt{2}a(\pi-2)-\sqrt{2}a]}$$

$$= \frac{5\pi-8}{2[9(\pi-2)-1]}$$ (5)

$$= \frac{5\pi-8}{2[9\pi-19]}$$

20

Part B (CM2)

17.

(a) $P(B) = 3, \quad P(B \cup C) = 0.37 \; and \quad P(C) = 0.2$

(i) $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ (05)

$(\therefore A \; and \; B \; are \; independent)$

$0.37 = P(A) + o.3 - P(A).0.3$ (05)

$0.07 = P(A) \times 0.7 \Rightarrow P(A) = 0.1$

(ii) $P(B' \backslash A^1) = \frac{P(B' \cap A')}{P(A')}$ (05)

$P(B' \cap A') = P(B \cup A)' = 1 - P(B \cup A)$

$= 1 - 0.37 = 0.63$ (05)

$P(A') = 1 - P(A) = 1 - 0.1 = 0.9$

$\therefore \; P(B' \backslash A') = \frac{0.63}{0.9} = 0.7$ (05)

(iii) $P(A' \cap B' \cap C) = P(A')P(B')P(C)$ (05)

$= 0.9 \times 0.7 \times 0.2$

$= 0.126$ (05)

(iv) $Let \; X = \; (A \cap B' \cap C') \cup P(A' \cap B \cap C') \cup (A' \cap B' \cap C)$ (05)

$\therefore P(X) = P(A \cap B' \cap C') + P(A' \cap B \cap C') + P(A' \cap B' \cap C)$

$= P(A) \; P(B') \; P(C') + P(A') \; P(B) \cup (C') + P(A')P(B')P(C)$ (05)

$= 0.1 \times 0.7 \times 0.8 + 0.9 \times 0.3 \times 0.8 + 0.9 \times 0.7 \times 0.2$

$= 0.398$ (05)

$\Rightarrow P(A/X) = \frac{P(A \cap X)}{P(X)}$

$= \frac{P(A \cap B' \cap C')}{P(X)}$ (05)

$= \frac{0.1 \times 0.7 \times 0.8}{0.398}$

$= \frac{28}{199}$ (05)

(b)

Distance	x_i	$y_i = \frac{x_i - 45}{10}$	f	fy	fy^2
0 – 10	05	-4	10	-40	160
10 – 20	15	-3	19	-57	171
20 – 30	25	-2	43	-86	172
30 – 40	35	-1	25	-25	25
40 – 50	45	0	8	0	0
50 – 60	55	1	6	6	6
60 – 70	65	2	5	10	20
70 – 80	75	3	3	9	27
80 – 90	85	4	1	4	16
			120	-179	597
	(05)	(05)		(05)	(05)

$\therefore\ y_i = \frac{x_i - 45}{10}$ (05)

$\therefore\ \bar{x} = 10\bar{y} + 45$

$Henc\ \bar{y} = \frac{\sum fy}{\sum f} = \frac{-179}{120} = -1.49$ (05)

$\therefore\ \bar{x} = 10(-1.49) + 45 = 30.08$ (05)

$\sigma y^2 = \frac{\sum f y_c^2}{\sum f} - \bar{y}^2$ (05)

$= \frac{1}{120}\ (597 - 120 \times 2.22)$ (05)

$= \frac{1}{120}\ (597 - 266.40) = 2.76$ (05)

(05)

$\sigma x^2 = 10^2 \sigma y^2 = 100 \times 2.76 = 276$ (05)

$\therefore\ \sigma x = 16.61$ (05)

65

11. $Number\ transfixed\ =\ 15$

$\therefore$ *The new distribution has only* 1^{st} … … … … … … … … … . . *total number*

$120 - 15 = 105$

1^{st} … … … … … … … … … … … . [10,20]

$\because\ Q_1 = \frac{1}{4} \times 105^{th}\ position =\ 26.25^{th}\ position$

$=\ 10\ +\ \frac{(26.25-10)}{19} \times 10$ (05)

$=\ 10\ + 8.55 = 18.55$ (05)

$3^{rd} Quater\ Q_3 = \frac{3}{4}(105)^{th}\ position$

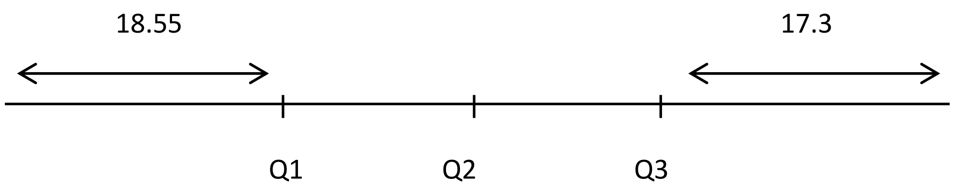
$= 78.75^{th}\ position$

The requared … … … … … … . *is* [30,40]

$Q_3 = 30 + \frac{(76.75-72)}{25} \times 10$ (05)

$= 30 + 2.7 = 32.7$ (05)

$\therefore IQR = 32.7 - 18.55 = 14.15$ (05)



$\therefore$ *The distribution is approximately Symmetric.* (05)

30